PERGAMON

International Journal of Heat and Mass Transfer 46 (2003) 687–694

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

The radiative transfer solution of a rectangular enclosure using angular domain discrete wavelets

Oguzhan Guven, Yildiz Bayazitoglu *

Department of Mechanical Engineering and Materials Science, Rice University, 6100 Main St. MS 321, Houston, TX 77005-1892, USA

Received 12 April 2002; received in revised form 31 July 2002

Abstract

The wavelet expansion is used in order to evaluate the angular dependence of the radiative intensity in the solution of the radiative transfer equation. The radiative intensity is expanded in terms of orthogonal Daubechies' wavelet basis in the angular domain. The method is applied to a two-dimensional rectangular enclosure with an absorbing, emitting and nonscattering medium in radiative equilibrium. One of the boundary surfaces is maintained at constant temperature T_1, while others are kept cold. This boundary conditions are chosen to demonstrate the effectiveness of the method in dealing with the geometries which are sensitive to ray effects. Centerline emissive power and surface heat flux distributions are compared well with the results given by the standard discrete ordinates method, the modified discrete ordinates method and also with the available exact solutions.

1. Introduction

Radiation is the dominant mode of heat transfer in many high-temperature systems and various methods have been proposed for the solution of the radiative transfer equation (RTE). These include Monte Carlo, zonal, spherical harmonics (P_N approximation), discrete ordinates (S_N), finite-volume methods [1,2]. The zonal and Monte Carlo methods are very accurate. However, it is now well established that both these methods are computationally intensive and are difficult to incorporate. The P_N methods have received much attention, yet the P_1 approximation is inaccurate and the formulation of higher order approximations is complicated and their implementation leads to important computational times without substantial gain in accuracy. Among others, the discrete ordinates method has had the most attention owing to its simple formulation, relatively good accuracy and compatibility with existing computer codes used in the transport processes involved in many convective transport problems. However, discrete ordinates predictions suffer from some shortcomings such as ray effects, occurrence of negative intensities during the solution process, and false scattering (a numerical phenomenon arising from the chosen spatial discretization scheme) [3]. Cheong and Song [4] incorporated cubic interpolation into the standard discrete ordinates method (SDO), considering numerical accuracy and grid dependence. They showed that discrete ordinates interpolation method (DOIM) mitigates the errors introduced by false scattering. Nonetheless, the DOIM does not improve the results in terms of ray effects. In order to deal with ray effects, Ramankutty and Crosbie [5] introduced modified discrete ordinates method (MDO), a semi-analytic method. They split the intensity into direct and diffuse components. The direct component is determined analytically, and the diffuse transport equation is solved numerically by conventional discrete ordinates procedure. MDO decreased the anomalies caused by the ray effects, yet there can be observed some anomalies for small aspect ratios. Besides, it is hard to handle anisotropic and other scattering models with MDO due to its analytic nature. The finite-volume

* Corresponding author. Tel.: +1-713-348-6279; fax: +1-713-348-5423.

E-mail address: bayaz@rice.edu (Y. Bayazitoglu).

0017-9310/03/$ - see front matter
PII: S0017-9310(02)00326-5

Nomenclature

$a_{m,n}$, $b_{m,n}$, $c_{m,n}$, $d_{m,n}$	wavelet expansion coefficients of radiative intensity
$A_{mm',nn'}$, $B_{mm',nn'}$	bookkeeping matrixes
E	emissive power ($=\sigma T^4$)
i, j, k, l	radiative intensity in four subdomains
I	radiative intensity
h	wavelet coefficient
L	distance between the plates for one-dimensional geometry
N	number of wavelet expansion terms
q	radiative heat flux
r	aspect ratio of two-dimensional rectangular enclosure
T	temperature
W_i	wrapped Daubechies wavelet functions
$W_{i,j}$	two-dimensional wrapped Daubechies wavelet functions
x, y, z	coordinates

Greek symbols

δ	Kronecker δ-function
ε	emissivity
ϕ	azimuthal angle measured from the positive x-axis
φ	dilation function
κ	absorption coefficient
μ, ξ	directional cosines
θ	polar angle measured from the positive z-axis
ρ	reflectivity
σ	Stefan–Boltzmann constant
τ_y, τ_y	optical thicknesses in y- and z-directions
ψ	wavelet function

Subscripts

m, m', n, n'	wavelet basis
1	hot wall
2	cold walls

Superscripts

t	transpose
$+, -$	positive and negative directions in one-dimensional case

method (FVM) [6,7] is a discrete ordinates type of method. The FVM can be regarded as the most sophisticated scheme among the currently available schemes. The main advantage of the FVM procedure is that a user has a complete flexibility in laying out the spatial and angular grids that best capture the physics of a given problem. However, there is a great complexity in extending its application to three-dimensional enclosures. Ray effects and false scattering are also encountered with the FVM.

In the past decade, the theory of wavelet analysis has been developed and applied to various fields such as signal processing, solution of partial differential equations [8] and integro-differential equations [9]. Wavelet analysis can be viewed as a multi-resolution analysis which consist of a sequence of successive approximation spaces. Donoho [10] showed that wavelets are unconditional bases for a very wide set of function classes. Particularly, when the functions exhibit localized variation, wavelets provide very good approximations.

Bayazitoglu and Wang [11] introduced the wavelet expansion into the solution of radiative transfer problems for nongray media. They used the wavelets in order to evaluate the spectral intensity function in frequency domain. In this work P_N approximation was used. Later, Wang and Bayazitoglu [12,13] replaced the P_N approximation with the discrete ordinates procedure.

The purpose of this article is to present a new numerical scheme which introduces the wavelets expansion in the evaluation of radiative intensity in angular domain. The intensity is expanded in terms of wavelet functions in angular domain. By doing so, the information related to directional distribution of the intensity field is stored in wavelets. This transforms the RTE to a new set of partial differential equations in terms of wavelet expansion coefficients which contains the frequency and spatial information of the intensity. This new set of partial differential equations are solved with finite differencing in spatial coordinates.

In this manuscript, first mathematical background on wavelets and details of the application of the method to the RTE is presented, then a two-dimensional test problem is discussed.

2. Mathematical analysis and formulation

In order to construct a wavelet function ψ, Daubechies [14] started from the dilation equation for the scaling function φ,

$$\varphi(x) = \sum_n h_n \varphi_{-1,n} = \sqrt{2} \sum_n h_n \varphi(2x - n) \quad n = 0, \quad N - 1 \tag{1}$$

and found that the wavelet function satisfies a similar dilation equation,

$$\psi(x) = \sqrt{2}\sum_n (-1)^{n-1} h_{N-n-1}\varphi(2x-n) \quad n = 0, \quad N-1 \tag{2}$$

More importantly, a set of h_n coefficients up to $N = 20$ (where N has to be even) is constructed. Since φ and ψ have finite support (i.e., they only have nonzero values in a finite interval), they are calculated numerically. Daubechies [14] proved that wavelets construct a set of orthogonal bases for the L^2 function space, and gave a detailed construction procedure for these wavelets. Newland [15] gave a wavelet series expansion of the L^2 function $f(t)$ as follows:

$$f(t) = b_0 + \sum_j \sum_k b_{2^j+k} W(2^j t - k), \quad 0 \leqslant t < 1,$$
$$j = 0, \infty, \ k = 0, 2^j - 1 \tag{3}$$

where $W(2^j t - k)$ are Daubechies' wavelets confined in the interval $0 \leqslant t < 1$ and wrapped around this interval as many times as necessary to ensure that their entire length is included in this interval; therefore, outside of this interval these wrapped around wavelets vanish to zero. The inner product of any single wavelet or any two distinct wavelets from the same family is identically zero. These orthogonality properties are expressed as

$$\int_0^1 W(2^j t - k) W(2^{j'} t - k')\,\mathrm{d}t = \delta_{jj'}\delta_{kk'} \tag{4a}$$

$$\int_0^1 W(2^j t - k)\,\mathrm{d}t = 0 \tag{4b}$$

where δ is the Kronecker δ function. The general coefficients can be calculated by taking the inner product of the function and the wavelet basis as

$$b_0 = \int_0^1 f(t)\,\mathrm{d}t \tag{5a}$$

$$b_{2^j+k} = \int_0^1 f(t) W(2^j t - k)\,\mathrm{d}t \tag{5b}$$

Newland [15] has developed a very efficient algorithm to compute the discrete wavelet transform Eqs. (5a) and (5b) from sampling points of the function. The wavelets W_m are calculated numerically from the inverse discrete wavelet transform.

Wavelet expansion can be applied to a two-dimensional function $f(x,y)$ function in a similar fashion to one-dimensional case (Eq. (3)) [15]:

$$f(x,y) = \mathbf{W}(x)\mathbf{C}\mathbf{W}^{\mathrm{t}}(y) \tag{6}$$

$\mathbf{W}(x)$ and $\mathbf{W}(y)$ are $1 \times N$ wavelet basis matrices, $\mathbf{C}$ is $N \times N$ wavelet coefficient matrix. N is the number of wavelet basis that are used for each dependent variable of the function. We can rearrange the two-dimensional expansion with a different notations in the following way:

$$f(x,y) = \sum_m^N \sum_n^N c_{m,n} W_{m,n}(x,y) \tag{7}$$

where $W_{m,n}(x,y)$ are the two-dimensional wavelets.

We consider radiative heat transfer in absorbing, emitting, nonscattering, two-dimensional rectangular media bounded by diffusely emitting and reflecting opaque walls. The radiative properties are assumed gray and spatially homogeneous. A schematic of the physical model and coordinates is illustrated in Fig. 1. The mathematical description to this problem is

$$\sin\theta\sin\phi\frac{\partial I(\tau_y,\tau_z,\theta,\phi)}{\partial\tau_y} + \cos\theta\frac{\partial I(\tau_y,\tau_z,\theta,\phi)}{\partial\tau_z}$$
$$+ I(\tau_y,\tau_z,\theta,\phi) = I_{\mathrm{b}}(\tau_y,\tau_z) \tag{8}$$

where $I(\tau_y,\tau_z,\theta,\phi)$ is the total intensity at the position (τ_y,τ_z) and in a direction which is expressed in terms of the *polar angle* θ and the *azimuthal angle* ϕ (see Fig. 1). $I(\tau_y,\tau_z,\theta,\phi) \in L^2(R)$ is bounded in the angular domain. I_{b} is the black body intensity.

$$I_{\mathrm{b}}(\tau_y,\tau_z) = \sigma T^4(\tau_y,\tau_z)/\pi \tag{9}$$

where σ is the Stefan–Boltzmann constant. $\tau_y = \kappa y$ and $\tau_z = \kappa z$ are optical thicknesses and κ is the absorption coefficient of the medium. Instead of the traditional definition of directional cosines, we prefer to use the following parameters: $\mu = \cos\theta$ and $\xi = \sin\phi$. The goal is to limit the number of angular parameters to two regardless of geometry being two- or three-dimensional. With these parameters, we will rearrange Eq. (8):

$$\xi\sqrt{1-\mu^2}\frac{\partial I}{\partial\tau_y} + \mu\frac{\partial I}{\partial\tau_z} + I = I_{\mathrm{b}} \tag{10}$$

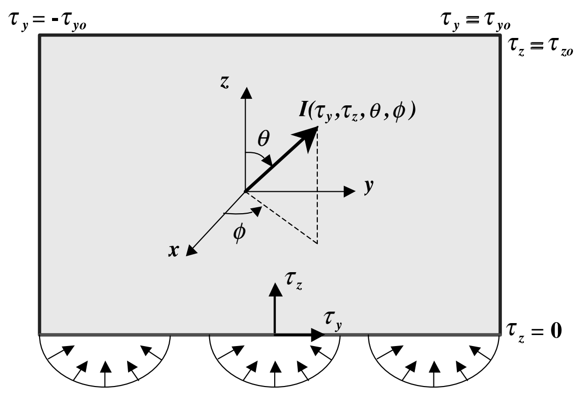

Fig. 1. Sub-domains for angular variation of the radiative intensity.

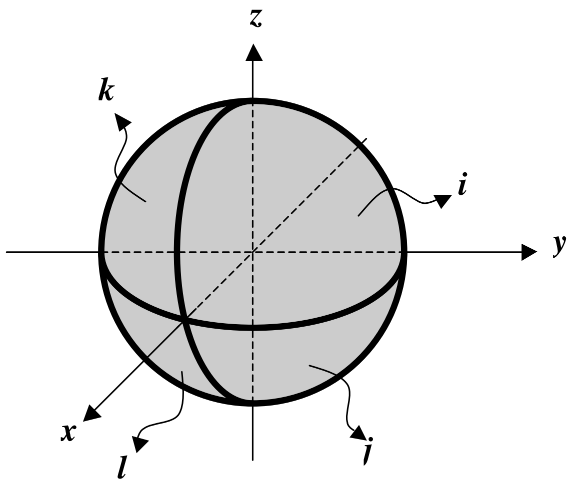

Fig. 2. Two-dimensional enclosure.

The procedure that we will follow requires the expansion of the intensity function $I = I(\tau_y, \tau_z, \theta, \phi)$ in angular domain (μ, ξ) into the wavelet series. We will use the Daubechies' wavelets and they have only finite support in [0,1]. However, μ and ξ have values from negative one to positive one. Therefore, we will divide the angular domain into four subdomains (Fig. 2), and denote the intensity I with i, j, k, l in these subdomains:

$$i = i(\tau_y, \tau_z, \mu, \xi) \quad \text{where } 0 \leqslant \mu < 1 \quad \text{and} \quad 0 \leqslant \xi < 1$$

$$j = j(\tau_y, \tau_z, \mu, \xi) \quad \text{where } -1 \leqslant \mu < 0 \quad \text{and} \quad 0 \leqslant \xi < 1$$

$$k = k(\tau_y, \tau_z, \mu, \xi) \quad \text{where } 0 \leqslant \mu < 1 \quad \text{and} \quad -1 \leqslant \xi < 0$$

$$l = l(\tau_y, \tau_z, \mu, \xi) \quad \text{where } -1 \leqslant \mu < 0 \quad \text{and} \quad -1 \leqslant \xi < 0$$

If we shift the directional cosines so that $0 \leqslant \mu < 1$ and $0 \leqslant \xi < 1$ for each subdomains, we can rewrite RTE in these subdomains as follows:

$$\xi\sqrt{1-\mu^2}\frac{\partial i}{\partial \tau_y} + \mu\frac{\partial i}{\partial \tau_z} + i = I_\mathrm{b} \qquad 0 \leqslant \mu < 1 \quad \text{and} \quad 0 \leqslant \xi < 1 \tag{11a}$$

$$\xi\sqrt{1-\mu^2}\frac{\partial j}{\partial \tau_y} - \mu\frac{\partial j}{\partial \tau_z} + j = I_\mathrm{b} \qquad 0 \leqslant \mu < 1 \quad \text{and} \quad 0 \leqslant \xi < 1 \tag{11b}$$

$$-\xi\sqrt{1-\mu^2}\frac{\partial k}{\partial \tau_y} + \mu\frac{\partial k}{\partial \tau_z} + k = I_\mathrm{b} \qquad 0 \leqslant \mu < 1 \quad \text{and} \quad 0 \leqslant \xi < 1 \tag{11c}$$

$$-\xi\sqrt{1-\mu^2}\frac{\partial l}{\partial \tau_y} - \mu\frac{\partial l}{\partial \tau_z} + l = I_\mathrm{b} \qquad 0 \leqslant \mu < 1 \quad \text{and} \quad 0 \leqslant \xi < 1 \tag{11d}$$

Boundary conditions for the above set of equations are given below:

$$i(\tau_y, 0, \mu, \xi) = \varepsilon_1 I_\mathrm{b}(\tau_y, 0) + \frac{\rho_1}{\pi}\int_0^1\int_0^1 \left[j(\tau_y, 0, \mu, \xi) + l(\tau_y, 0, \mu, \xi)\right]\frac{\mu}{\sqrt{1-\xi^2}}\,\mathrm{d}\mu\,\mathrm{d}\xi, \qquad -\tau_{y0} < \tau_y < \tau_{y0} \tag{12a}$$

$$i(-\tau_{y0}, \tau_z, \mu, \xi) = \varepsilon_2 I_\mathrm{b}(-\tau_{y0}, \tau_z) + \frac{\rho_2}{\pi}\int_0^1\int_0^1 \left[k(-\tau_{y0}, \tau_z, \mu, \xi) + l(-\tau_{y0}, \tau_z, \mu, \xi)\right]\frac{\mu}{\sqrt{1-\xi^2}}\,\mathrm{d}\mu\,\mathrm{d}\xi, \qquad 0 < \tau_z < \tau_{z0} \tag{12b}$$

where ε, ρ are emissivity and reflectivity of the boundaries. Similar expressions for j, k and l could be written. For brevity, we will not include those in this paper. As it can be seen from above equations, a singularity occurs in the integration. These singularities are on the limits of the integrations. Hence, they can be treated with the methods dealing with the singularities of this type [16]. Wavelet series expansions of the intensities i, j, k, l are

$$i = \sum_m \sum_n a_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi) \tag{13a}$$

$$j = \sum_m \sum_n b_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi) \tag{13b}$$

$$k = \sum_m \sum_n c_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi) \tag{13c}$$

$$l = \sum_m \sum_n d_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi) \tag{13d}$$

where $a_{m,n}$, $b_{m,n}$, $c_{m,n}$ and $d_{m,n}$ are wavelet expansion coefficients, and $W_{m,n}$ are the two-dimensional wavelets. As it can be seen from Eqs. (13a)–(13d), the wavelet coefficients are only the function of position, and all the information related to the directional distribution of the intensities is packed in the wavelets. Now, we will insert the Eqs. (13a)–(13d) into Eq. (11a).

$$\xi\sqrt{1-\mu^2}\frac{\partial}{\partial \tau_y}\left[\sum_m \sum_n a_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi)\right] + \mu\frac{\partial}{\partial \tau_z}\left[\sum_m \sum_n a_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi)\right] + \sum_m \sum_n a_{m,n}(\tau_y, \tau_z) W_{m,n}(\mu, \xi) = I_\mathrm{b}(\tau_y, \tau_z) \tag{14}$$

Similar expressions can be obtained for Eqs. (11b)–(11d). We will apply the Galerkin method to the above equation. The weighting functions are chosen to be the same functions as the wavelet bases. After multiplying an individual wavelet on both sides and integrating in the angular domain, and using the following orthogonality properties of wavelets

$$\int_{\mu=0}^{1}\int_{\xi=0}^{1} W_{m,n}(\mu,\xi)W_{m',n'}(\mu,\xi)\,\mathrm{d}\xi\,\mathrm{d}\mu = \begin{cases} 1 & \text{if } m=m' \text{ and } n=n' \\ 0 & \text{otherwise} \end{cases} \tag{15a}$$

$$\int_{\mu=0}^{1}\int_{\xi=0}^{1} W_{m,n}(\mu,\xi)\,\mathrm{d}\xi\,\mathrm{d}\mu = \delta_{m,1}\delta_{n,1} \tag{15b}$$

we obtain following equations for four subdomains:

$$\sum_m\sum_n\left[A_{mm',nn'}\frac{\partial a_{m,n}(\tau_y,\tau_z)}{\partial\tau_y} + B_{mm',nn'}\frac{\partial a_{m,n}(\tau_y,\tau_z)}{\partial\tau_z}\right] + a_{m',n'}(\tau_y,\tau_z) = I_b(\tau_y,\tau_z)\delta_{m',1}\delta_{n',1} \tag{16a}$$

$$\sum_m\sum_n\left[A_{mm',nn'}\frac{\partial b_{m,n}(\tau_y,\tau_z)}{\partial\tau_y} - B_{mm',nn'}\frac{\partial b_{m,n}(\tau_y,\tau_z)}{\partial\tau_z}\right] + b_{m',n'}(\tau_y,\tau_z) = I_b(\tau_y,\tau_z)\delta_{m',1}\delta_{n',1} \tag{16b}$$

$$\sum_m\sum_n\left[-A_{mm',nn'}\frac{\partial c_{m,n}(\tau_y,\tau_z)}{\partial\tau_y} + B_{mm',nn'}\frac{\partial c_{m,n}(\tau_y,\tau_z)}{\partial\tau_z}\right] + c_{m',n'}(\tau_y,\tau_z) = I_b(\tau_y,\tau_z)\delta_{m',1}\delta_{n',1} \tag{16c}$$

$$\sum_m\sum_n\left[-A_{mm',nn'}\frac{\partial d_{m,n}(\tau_y,\tau_z)}{\partial\tau_y} - B_{mm',nn'}\frac{\partial d_{m,n}(\tau_y,\tau_z)}{\partial\tau_z}\right] + d_{m',n'}(\tau_y,\tau_z) = I_b(\tau_y,\tau_z)\delta_{m',1}\delta_{n',1} \tag{16d}$$

where $A_{mm',nn'}$ and $B_{mm',nn'}$ are defined as

$$A_{mm',nn'} = \int_{\mu=0}^{1}\int_{\xi=0}^{1}(\xi\sqrt{1-\mu^2})W_{m,n}(\mu,\xi)W_{m',n'}(\mu,\xi)\,\mathrm{d}\xi\,\mathrm{d}\mu \tag{17a}$$

$$B_{mm',nn'} = \int_{\mu=0}^{1}\int_{\xi=0}^{1}\mu W_{m,n}(\mu,\xi)W_{m',n'}(\mu,\xi)\,\mathrm{d}\xi\,\mathrm{d}\mu \tag{17b}$$

and can be readily calculated. In other words, they are known coefficient matrices of the differential Eqs. (16a)–(16d). The boundary conditions for Eq. (16a) in its most general form can be written as

$$a_{m',n'}(\tau_y=-\tau_{y0},\tau_z) = \left\{\varepsilon_1 I_b(\tau_y=-\tau_{y0},\tau_z) + \frac{\rho_1}{\pi}\sum_m\sum_n\left[c_{m,n}(\tau_y=-\tau_{y0},\tau_z) + d_{m,n}(\tau_y=-\tau_{y0},\tau_z)\right] \times \int_0^1\int_0^1 W_{m,n}(\mu,\xi)\frac{\xi\sqrt{1-\mu^2}}{\sqrt{1-\xi^2}}\,\mathrm{d}\mu\,\mathrm{d}\xi\right\}\delta_{m',1}\delta_{n',1} \tag{18a}$$

$$a_{m',n'}(\tau_y,\tau_z=0) = \left\{\varepsilon_1 I_b(\tau_y,\tau_z=0) + \frac{\rho_1}{\pi}\sum_m\sum_n\left[b_{m,n}(\tau_y,\tau_z=0) + d_{m,n}(\tau_y,\tau_z=0)\right]\int_0^1\int_0^1 W_{m,n}(\mu,\xi) \times \frac{\mu}{\sqrt{1-\xi^2}}\,\mathrm{d}\mu\,\mathrm{d}\xi\right\}\delta_{m',1}\delta_{n',1} \tag{18b}$$

by utilizing the wavelet expansion method. Similar expressions can be obtained for Eqs. (16b)–(16d).

With the help of the procedure explained above, the RTE has been converted to a new set of partial differential equations written in terms of wavelet coefficients. There are many types of numerical differencing schemes to solve Eqs. (16a)–(16d). Here, we chose a finite differencing method with the step scheme. We skip the details of this procedure.

In order to complete the solution method for a radiative heat transfer system, we need to incorporate the energy equation into the solution procedure. This is achieved by the modified quasi-linearization algorithm. Details of this can be found in Refs. [13,18]. In this work, we assumed radiative equilibrium and energy equation in this case is $\Delta\boldsymbol{q}=0$. After solving Eqs. (16a)–(16d) for the wavelet expansion coefficients a, b, c and d, heat fluxes are calculated with the following expression:

$$q_y(\tau_y,\tau_z) = 2\sum_m\sum_n\left[a_{m,n}(\tau_y,\tau_z) + b_{m,n}(\tau_y,\tau_z) - c_{m,n}(\tau_y,\tau_z) - d_{m,n}(\tau_y,\tau_z)\right] \times \int_0^1\int_0^1 W_{m,n}(\mu,\xi)\frac{\xi\sqrt{1-\mu^2}}{\sqrt{1-\xi^2}}\,\mathrm{d}\mu\,\mathrm{d}\xi \tag{19a}$$

$$q_z(\tau_y,\tau_z) = 2\sum_m\sum_n\left[a_{m,n}(\tau_y,\tau_z) - b_{m,n}(\tau_y,\tau_z) + c_{m,n}(\tau_y,\tau_z) - d_{m,n}(\tau_y,\tau_z)\right] \times \int_0^1\int_0^1 W_{m,n}(\mu,\xi)\frac{\mu}{\sqrt{1-\xi^2}}\,\mathrm{d}\mu\,\mathrm{d}\xi \tag{19b}$$

Again the singularity in the integration is treated with the methods explained in [16].

3. Results

The wavelet method is applied to a two-dimensional rectangular enclosure containing an absorbing, emitting and nonscattering medium as illustrated in Fig. 1. All the surfaces are assumed to be black and isothermal. The bottom wall is exposed to a diffuse loading ($T_1=1$), and no loading is applied on the other walls ($T_2=0$). For this system, aspect ratio is defined as $r=2\tau_{y0}/\tau_{z0}$. This test problem is chosen because the discrete ordinates type of methods are susceptible to ray effects for the specified boundary conditions. However, the wavelet method is immune from these effects since it fully models the angular distribution of the intensity at every point through the spatial domain. We illustrate this with an example: Let's consider an absorbing, emitting and nonscattering, plane-parallel medium. (Here, we choose this geometry as the example case because analytical solution to the intensity field is available for the one-dimensional problems [1].) The left wall is hot while the right one is kept cold. Fig. 3 shows the intensity field at a point half way through the distance between the plates, L.

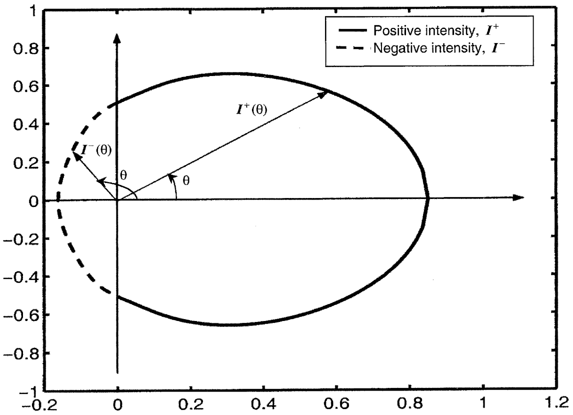

Fig. 3. Intensity field at the midpoint of the plane-parallel geometry.

The error in the approximate intensity field as compared with the exact solution [1] at $x = L/2$ at polar angles $\theta = 0°$, 51.7553°, 81.7843° and 159.4713° are 0.53%, 0.72%, 0.32% and 1.79%, respectively. This proves that the present method successfully approximates the angular distribution of the radiative intensity. After obtaining intensity distribution in angular domain at each node of a spatial grid, the method uses a finite differencing scheme between these approximations.

The nondimensional emissive power at the centerline and the heat flux results at the top wall are presented in Figs. 4–8. They are compared with exact results [17], and with those of the standard discrete ordinates (SDO-S_8) [5] and the modified discrete ordinates (MDO-S_8) [5] methods. The MDO has been developed to mitigate the ray effects in SDO results and it has been successful to

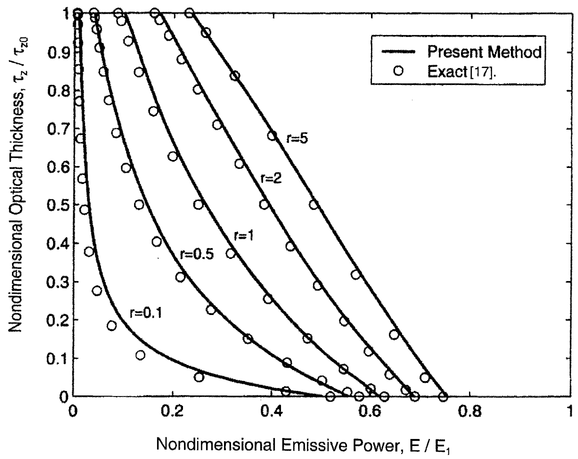

Fig. 4. Comparison of the centerline nondimensional emissive power results for different aspect ratios with exact results [17], $\tau_{y0} = 0.5$.

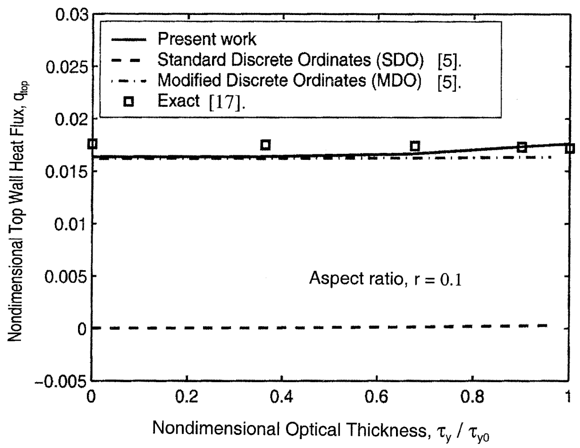

Fig. 5. Comparison of the nondimensional surface heat flux results given by wavelet, the SDO [5], the MDO [5] methods and exact solution [17] at the top wall for $r = 0.1$ and $\tau_{z0} = 1$.

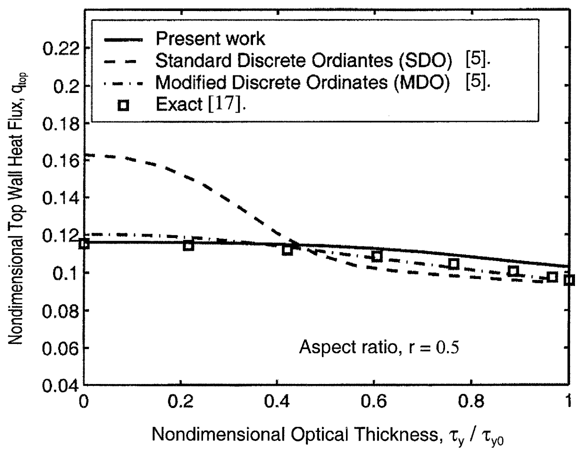

Fig. 6. Comparison of the nondimensional surface heat flux results given by wavelet, the SDO [5], the MDO [5] methods and exact solution [17] at the top wall for $r = 0.5$ and $\tau_{z0} = 1$.

some extent. However, it is important to state that it is a semi-analytical method. This analytic nature is a setback in terms of the full automation of the solution method. Besides, it presents difficulties in handling radiation problems that involve anisotropic and other types scattering phase functions [5].

The nondimensional emissive power results for different aspect ratios are plotted in Fig. 4 and compared with the exact results [17]. Emissive power results display a good agreement with the exact results. Figs. 5–8 present the surface heat flux results at the top wall. For all the aspect ratio values, the surface heat flux results of the SDO are inaccurate. They display oscillations especially for small aspect ratios. The MDO agrees well with the exact results for all the aspect ratios except for $r = 0.1$.

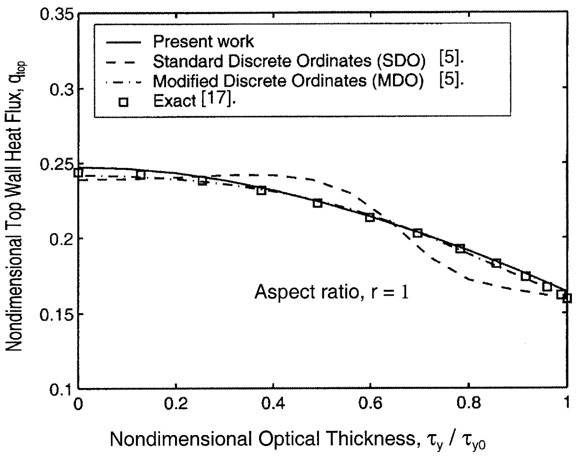

Fig. 7. Comparison of nondimensional surface heat flux results given by wavelet, the SDO [5], the MDO [5] methods and exact solution [17] at the top wall for $r = 1$ and $\tau_{z0} = 1$.

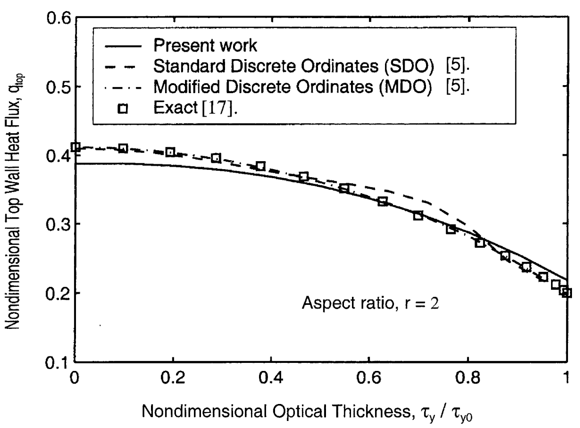

Fig. 8. Comparison of nondimensional surface heat flux results given by wavelet, the SDO [5], the MDO [5] methods and exact solution [17] at the top wall for $r = 2$ and $\tau_{z0} = 1$.

Ramankutty and Crosbie [5] attributed the presence of anomalies at the heat flux results in the case of $r = 0.1$ to the inability of the MDO to remove the ray effects completely. The present method exhibits good agreement with exact results for all the aspect ratio cases. The errors of the present method do not show dependence to aspect ratio. This might be explained as follows: Wavelet bases are very capable of approximating functions with local changes. Therefore the wavelet method does not suffer from the ray effects. However, the finite differencing scheme which is used to solve the Eqs. (16a)–(16d) is incapable of completely capturing sharp temperature and intensity gradients through the spatial domain, and causes them to be smoothened out. As it can be seen from Figs. 5–8, the surface heat flux profiles are somewhat flattened for all cases. Hence, the utilization of a better spatial discretization method could improve the results.

The code when a 20×20 spatial discretization and eight wavelet expansion coefficients are used converges in 15 s for $r = 0.1$, 25 s for $r = 1$ and 30 s for $r = 2$ on a SPARC Ultra-4 machine.

4. Conclusion

The present method is fully numerical. It utilizes the wavelet bases to model intensity field in angular domain. In this work Daubechies' orthogonal wavelet bases are used. However, any other orthogonal wavelet can be used in the same way. This process converts the RTE into a set of partial differential equations (Eqs. (17a) and (17b)) only in terms of position. This set can be handled with any spatial discretization method. In this work finite differencing with step scheme is used. The results agree fairly well with the exact results. They might be improved by introducing better spatial discretization methods. The method is flexible to handle the radiative transfer problems with an anisotropic scattering medium. The application of the method to a problem with a linear anisotropic medium is demonstrated in [19].

References

[1] M.F. Modest, Radiative Heat Transfer, McGraw-Hill, New York, 1993.

[2] R. Siegel, J.R. Howell, Thermal Radiation Heat Transfer, Hemisphere Publishing Corporation, Washington, DC, 1992.

[3] J.C. Chai, H.S. Lee, S.V. Patankar, Ray effect and false scattering in the discrete ordinates method, Numer. Heat Transfer, Part B 24 (1993) 373–389.

[4] K.B. Cheong, T.H. Song, An alternative discrete ordinates method with interpolation and source differencing for two-dimensional radiative transger problems, Numer. Heat Transfer, Part B 32 (1997) 107–125.

[5] M.A. Ramankutty, A.L. Crosbie, Modified discrete ordinates solution of radiative transfer in two-dimensional rectangular enclosures, J. Quantum Spectrosc. Radiat. Transfer 57 (1) (1997) 107–140.

[6] G.D. Raithby, E.H. Chui, A finite-volume method for predicting a radiant heat transfer in enclosures with participating media, ASME J. Heat Transfer 112 (1990) 415–423.

[7] J.C. Chai, H.S. Lee, S.V. Patankar, Finite volume method for radiation heat transfer, J. Thermophys. Heat Transfer 8 (3) (1994) 419–425.

[8] K. Amaratunga, J.R. Williams, S. Qian, J. Weiss, Wavelet-galerkin solution for one-dimensional partial differential equations, Int. J. Numer. Meth. Eng. 37 (1994) 2703–2716.

[9] A. Avudainayagam, C. Vani, Wavelet-galerkin method for integro-differential equations, Appl. Numer. Math. 32 (2000) 247–254.

[10] D.L. Donoho, Unconditional bases are optimal bases for data compression and for statistical estimation, Appl. Comput. Harmonic Anal. 1 (1) (1993) 100–115.

[11] Y. Bayazitoglu, B.Y. Wang, Wavelets in the solution of nongray radiative heat transfer equation, J. Heat Transfer 120 (1998) 133–139.

[12] Y. Wang, Y. Bayazitoglu, Wavelets and discrete ordinates method in solving one-dimensional nongray radiation problem, Int. J. Heat Mass Transfer 42 (1999) 385–393.

[13] Y. Wang, Y. Bayazitoglu, Wavelets and discrete ordinates method for radiative heat transfer in a two-dimensional rectangular enclosure with a nongray medium, J. Heat Transfer, in press.

[14] I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, PA, 1992.

[15] D.E. Newland, An Introduction to Random Vibrations, Spectral & Wavelet Analysis, John Wiley and Sons, New York, 1993.

[16] W. Squire, Integration for Engineers and Scientists, American Elsevier Publishing Company, New York, 1970.

[17] A.L. Crosbie, R.G. Schrenker, Radiative transfer in a two-dimensional rectangular medium exposed to diffuse radiation, J. Quant. Spectrosc. Radiat. Transfer 31 (4) (1984) 339–372.

[18] A. Miele, R.R. Iyer, General technique for solving nonlinear two-point boundary-value problems via the method of particular solutions, Optim. Theory Appl. 5 (1970) 382.

[19] O. Guven, B.Y. Wang, Y. Bayazitoglu, Solving radiative transfer equation in scattering plane-parallel medium using wavelets approximation, in: Proceedings of ASME International Mechanical Engineering Congress and Exposition, IMECE 2002-3388, New Orleans, Louisiana, 2002.